Clabacus: A Risk-Adjusted Cloud Resources Pricing Model Using Financial Option Theory

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Abstract—In Cloud computing, clients would like to pay fair price for the resources while providers would like to make profit for their services. In this study, we propose a Cloud Compute Commodity (C³) pricing architecture called Clabacus(Cloud-Abacus) to serve both parties. We use concepts and algorithms from financial option theory to develop Clabacus. We propose a general formula, called compound-Moore’s law, that captures the technological advances of the resources, rate of inflation and depreciation etc. We map these Cloud parameters to the option pricing parameters to effectively modify the option pricing algorithm in order to compute Cloud resource price. Using financial value-at-risk (VaR) analysis, we adjust the computed resource price to incorporate the inherent risks of the Cloud provider. We propose fuzzy logic and genetic algorithm based approaches to compute the VaR of the provider’s resources. We have incorporated this into our Clabacus architecture. Finally, we study the effects of quality of service, rate of depreciation, rate of inflation, capital investment on the Cloud resource price for both client and provider. We show that if the prices are adjusted within a lower and upper bound, SLA can be guaranteed.

Index Terms—Resource pricing, Clabacus, financial options, value-at-risk, fuzzy logic, genetic algorithm

1 INTRODUCTION AND MOTIVATION

Over the past few years Cloud computing has dominated the IT industry and academia. One of the important reasons for this increased popularity is its ease and accessibility of shared resources. With Cloud, IT professionals profit by allowing the shared resources allocated as per demand or using a pay-per-use model [1]. Examples of popular Cloud services that use this model include Amazon Web Services, Google Apps and VMWare vCloud.

As per the 2011 readership survey by TheServerside.com (see http://www.theserverside.com/feature/Going-Public-Top-3-Public-Cloud-Providers-for-Business) (last accessed on May 18, 2014), The Amazon EC2 is the preferred option for a full 60 percent of respondents. It is currently geared towards medium to large-sized businesses. However, the introduction of micro-instances has helped target small businesses as well. The Google App development system commands a respectable 35 percent of the market share for public Cloud business users, in particular for small businesses. VMWare’s vCloud has recently become popular.

The Cloud computing literature has been witnessing a lot of research efforts on resource virtualization, resource scheduling/provisioning, data management and migration, and security (see for example [2], [3], [4], [5], [6]). While these and many current on-going studies strive to provide seamless service to clients’ demands at high quality, the fee for availing these services has been decided by the providers. The cost for such services are in general nominal, which for an occasional user may not be a big burden; however, businesses that avail of the Cloud services on a continuous basis at a certain fee might be incurring more expenses in using these services compared to owning the required infrastructure itself. For infrastructure owners profiting from their capital investments (initial and recurring) in a timely manner is an essential step to remain in business. Timeliness of the profits is even more important in Cloud due to continuous infrastructure upgrade required to remain competitive in Cloud provisioning. They should have robust model(s) for pricing the resources with profitability in one hand and market competition on the other. Therefore, resource pricing is an important problem in Cloud for both the providers and clients.

1.1 Financial Options and Cloud Resource Pricing

The compute resources in Cloud exist as commodities distributed across geographical regions. In this paper, we use the term Cloud Compute Commodities (C³) to address the Cloud resources, which may include CPU, bandwidth, storage etc. Cloud servers are provided, for example Amazon, in three instances: reserved, on-demand and spot. For the reserved services the negotiation on price may still exist, it does not provide much option than paying whatever is asked by the providers. On other two instances the possibility of negotiation is very good and hence several optionality of service contracts exists. By “optionality” we mean the flexibility for the clients in the service contracts. Since the contracts in Cloud services do not follow simple supply-demand models, we have to resort to finance based models such as option pricing. Some economic principles based

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model such as Net Present value (NPV), Discounted Cash Flow [7], [8], [9] do not capture important features of the flexibility and negotiability of service contracts and hence financial option pricing becomes an important parallel to the C³ pricing. In addition, economic based model for pricing do not capture the technological evolution in the computing world to account for the depreciation of the resources, where as with financial model it becomes easy to integrate technology with financial principles to remain profitable and also competitive in the market. All these reasons are the impetus for the use of financial option principles to pricing C³.

Financial options form a contract between two parties [10]. Financial options are of two types: call options and put options. A call/put option gives the holder (client) the right to buy/sell an underlying asset (such as a stock (Cloud resource)) at a future date at a price (the strike price) specified at the time of writing the option. The writer (Cloud service provider) of the option is obliged to the decision of the holder. These two types of options are exercised in the market in different styles. In this paper, we use one simple style of option, the European option. A European option grants the holder the right to exercise the option only at the maturity date.

In this paper, we map each of the Cloud resources as an individual asset in the option contract and model the problem of resource pricing as an option pricing problem.

### 1.2 Mapping C³ and Contribution

Three major parallels between financial option and C³ are:

1) Pricing of Cloud resources is still in its early stages and researchers are looking for effective ways to price them. One of the major contributions of this research is mapping Cloud resource pricing to well-established financial option pricing models and apply them to compute prices for the Cloud resources. This mapping is discussed in Section 3 with the aid of one of the intuitive models, binomial lattice, in Section 4.

2) Another major research area in finance is estimating the risk associated with investments and it requires intricate mathematical formulation and heuristics. The financial risk associated with the Cloud data center is not well understood and the second major contribution of this work is to apply the risk estimation models from finance to Cloud computing. We propose fuzzy logic and genetic-algorithm (GA) approaches for evaluating value-at-risk (VaR) for data centers as explained in Section 5. In order for the data center to remain profitable, resource prices computed using option pricing techniques will be adjusted knowing the VaR for the data center.

3) In Cloud computing, counter-party risk is common. That is, service providers who do not own resources are in such vulnerable position of breaching the service level agreements (SLA) with the clients if and when infrastructure owner defaults on his/her responsibility of providing resources to the service provider. The third contribution of the research is to use the principles of Collateral Service Agreements (CSA) concepts in finance markets to refine the SLA in Cloud contracts. Studying the effects of parameters such as the inflation rate, start time and use time of the resource, quality of service, rate of depreciation of the resource and capital investments on Cloud resource price helps providing a sound SLA guarantee. This is discussed in Section 6.

In summary, we bring the knowledge from computational finance to C³ pricing in a novel way. We propose to capture the Cloud market phenomenon in to a new framework called Clabacus as explained in Section 3.

The rest of this paper is organized as follows. We collect in Section 2 related work and organize them into two sections: cost optimization models and financed based models. We propose the Clabacus pricing architecture in Section 3. To keep the length of the manuscripts within a limit, in Section 4 we describe the computational module of the Clabacus architecture focusing on the binomial lattice algorithm, which is described in appendix, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TCC.2014.2382099.

We integrate the Moore’s law [11] into our study and mapping of Cloud parameters to the option pricing problem is achieved in this section. We discuss the Value-at-Risk analysis in Section 5 and describe how the output of the Cloud resource pricing algorithm in the previous section is used to adjust the resource prices for the risks involved in providing services. Experiments and results are discussed in Section 6 from both the provider and clients’ perspective. We establish the lower and upper bounds for the prices first. We conclude our study in Section 7 with a glimpse of future directions.

### 2 RELATED WORK

#### 2.1 Cost and Optimization

Economic pricing models such as Net present value and discounted cash flow [7] cannot incorporate the elasticity of Cloud resources. In [12], the author analyzes the true cost of leasing a CPU for an hour against acquiring and owning the same CPU. This study concludes that financial option based pricing would be an appropriate technique for Cloud resource pricing.

Patel and Shah [13] explore the cost incurred by data centres. This study focuses on three major issues: space, power and cooling on cost model. They provide a step by step analysis of the cost for each of the three issues and sum these costs to obtain a comprehensive cost of running a data centre. The authors of this study do not go any further in finding the cost of Cloud resources meant to be sold as a service.

Studying one year of Amazon’s spot instance price data the author in [14] concluded that by moving workload from off-peak periods to spot instances and scheduling for dispatching tasks additional cost savings could be achieved. In [15] the authors explored ways of increasing the profit for IaaS providers by increasing the resource utilization. They have studied this problem for a Cloud provider within a “Cloud federation” and suggested several policies to increase utilization based on the resource prices at other providers within the federation.
The authors in [16] have done a comprehensive analysis of one year spot instances price history in four data centres of Amazon's EC2. They showed that the statistical model they have proposed fits well with these data series and claim that they would be able to model the dynamics of spot price. This model could be used to predict the spot instance prices of Amazon EC2 instances.

Profit maximization and cost minimization are driving factors in the Cloud business, like in any other businesses. In [17] an attempt is made to optimize the profit from Cloud services through proper resource scheduling without violating the SLA constraints. Cost based scheduling is shown to do better than first come first serve or shortest job first approaches. By developing a simple workflow engine, a scheduling algorithm based on GA and PSO is proposed in [18] to optimize the workflow execution. They showed that their proposed GA-PSO algorithm minimizes the overall cost. In [19], the authors answer the question: which of the two services reserved or on-demand would bring higher revenue to the Cloud providers. Optimization of resource provisioning cost is studied in [20] where the authors propose four different statistical approaches for the problem and include demand and price uncertainty consideration and claim that their optimal Cloud resource provisioning algorithm minimize the total cost of resource provisioning. Cost minimization has been the focus in the study [21] using stochastic programming.

In [22] the authors have proposed a centralized decision based algorithm that adopts a game-theory approach to provide service to clients through cooperation as well as competition among the providers. Authors in [23] have proposed a simulator with pricing mechanism for the providers that considers flexibility in the requirement level of clients and hence flexible prices for the clients.

Considering provider-client system as a social system, welfare of the society is studied through Cloud bandwidth reservation pricing in [24]. Their trading system allows reservation of bandwidth for a variety of time durations at a cost optimal levels. A model for bandwidth allocation satisfying the client’s demands for both bandwidth and time is proposed in [25]. The authors claim that this model allows the flexible demands from the clients with differential pricing and maximizing the revenue for the providers.

Most of the above studies are based on static charges to the clients for the resource usage. A market driven dynamic pricing mechanism is proposed in [26] and revenue maximization for the providers is studied using dynamic programming. A similar study is presented in [27] for IaaS Cloud markets. In [28] a dynamic pricing model to get max profits and cost minimization using multi-constraint hybrid system is presented for PaaS in Cloud. Mihailescu and Teo [29] introduced a dynamic strategy-proof pricing scheme to incorporate diverse resource requests on a federated Cloud. Teng and Magoules [30] build dynamic billing and allocation policies that allow the user to predict future Cloud resource prices and adjust their budget accordingly.

Based on an idealistic situation of truth revelation of the needs from a customer Wu et al. [31] suggested option like contracts in IT provisioning, where they optimize the resource utilization and coordinator’s profits. Also, using option contracts improved infrastructure utilization and hedging cost of the energy required for running a Cloud data center were studied in [32], [33].

All these studies have focused on investigating the existing prices or on how to derive cost savings for the users based on current prices mostly for reserved and on-demand customers. To the best of our knowledge, devising a quantitative approach to price Cloud resources that would be profitable for the providers and competitive for the clients, has not been the subject of extensive investigation.

2.2 Pricing Cloud Resources

Macias and Guitart [34] use genetic algorithms to price Cloud resources based on the underlying rule that the prices of the Cloud resources may fluctuate based on their usage. In [35], the authors have surveyed current pricing plans for storage by few major Cloud service providers. They have separated the plans on a pointwise basis and on the overall basis. They observed that with the exception of Amazon, all Cloud providers use a bundling policy. The authors applied Parteo dominance analysis to short list the providers on the basis of price only.

In the recent past financial option concept has been explored for pricing Grid resources [36]. Due to price fluctuation in Cloud instances as well as fluctuation in their availability, the financial option concept used in [36] cannot be extended to price Cloud resource. A financial option based market model for federated Cloud has been proposed in [37]. In [38], the authors have explored financial option theory for pricing Cloud resources.

In the current study we propose a pricing architecture called Clabacus. In Clabacus, as a first step, we employ a risk-neutral (theory) based option pricing algorithms to price resources. Then, we compute the risk-adjusted resource prices by studying the value of the infrastructure investment at risk. This is a unique, fundamental and quantitative means for Cloud resource pricing and we show that the price generated from our architecture brings profit to the provider while at the same time competitive for the clients. Hence, this work contributes to Cloud computing in many fronts.

3 Clabacus Architecture

3.1 The Clabacus

The Fig. 1 presents Clabacus (Cloud-Abacus) architecture with various modules that work independently and collaboratively to compute the price of the resources. The explanation of each module follows.

Input module. This is the graphical user interface (GUI), where the Cloud resource provider will enter the various initial and normal recurring costs; including but not limited to hardware, land lease, energy (electricity, natural gas etc.), high quality personnel and insurance costs. To include other parameters affecting the cost, we have an additional input field for miscellaneous costs.

Input modifier module. This module converts various inputs into one standard unit and one currency. For example, some costs could be in $/hour (electricity), while some could be in £/year (land lease).

Driver. This is the main command center of Clabacus. The prime responsibility of the driver is to select the suitable
evaluation method from among many computational algorithms in the computation block based on the user inputs. The second task of the driver is to compute VaR using the value-at-risk module. The final task of the driver is to assemble the results from computation and VaR modules, adjust the price for risks before sending it to the output module.

Mapper. The computation algorithms use varying set of inputs to compute the price of Cloud resources. The task of mapper is to fine-tune the input parameters received from driver and feed it to the selected computational algorithm.

Computation block. This block includes various computation algorithms such as Black-Scholes-Merton closed form formula for option price, binomial lattice, Monte-Carlo (MC), finite-differencing, fast Fourier transform etc. In the current study we use binomial-lattice algorithm to price cloud resources.

Output. This is a graphical user interface and all the outputs will be displayed here.

4 CLABACUS: COMPUTATION BLOCK

There many option pricing algorithms available in the computation block. One such algorithm is binomial lattice model, which we describe next. We also explain in this section integration of Moore’s law with the binomial lattice algorithm. These are two fundamental and essential concepts for pricing the resources using Clabacus and form the basis of our computational model known as Compounded-Moore’s law. We also discuss in this section the mapping of Cloud pricing parameters to the binomial lattice algorithm.

4.1 Binomial Lattice Model

The binomial lattice model proposed by Cox et al. [39] is a popular numerical approach for option pricing in financial markets. The price movement of a stock is constructed as a tree with up and down movements. That is, the price of an asset (for example, a stock) can go up by a factor \( u \) or go down by a factor \( d \). These price changes are captured in the nodes of a binomial tree. \( u \) and \( d \) were shown to be evaluated from the volatility \( \sigma \) of the asset using \( u = e^{\sigma \sqrt{t}} \) and \( d = e^{-\sigma \sqrt{t}} \).

Let \( S \) be the current stock price, the price at the next time step could go up to \( S_u = S \times u \) or it can go down to \( S_d = S \times d \). This is repeated until the end of the contract period. The local pay-off at each of the leaf nodes is calculated as \( f_u = \text{Max}[S_u - K, 0] \) and \( f_d = \text{Max}[S_d - K, 0] \), and retain only positive values of pay-off. The objective is to evaluate the option price at the root node, the start time of the contract, so as to decide if it is worthwhile signing the SLA for the Cloud service. If \( p \) and \( (1 - p) \) are the probabilities for the asset price to go up and down respectively, the pay-off \( f_j \) at a node of the tree can be computed as the weighted sum of the pay-off \( f_u \) and \( f_d \) at two children nodes in the next step: \( f = pf_u + (1 - p)f_d \). Since this pay-off \( f_j \) is based on future values, it has to be discounted to find the value at the current time. Multiplying the weighted sum formula above with the discounting factor \( e^{-rT} \) [39], will give the current value of the option, where \( r \) is the interest rate. Tracing back the tree from the leaf nodes to the root node, the pay-off at the root node can be computed as

\[
f = e^{-rT} [pf_u + (1 - p)f_d]
\]

where, \( p = \frac{e^{\Delta t} - d}{u - d} \).

The accuracy of the results increase with the increase in the number of time steps. With large number of time steps, the results from binomial lattice algorithm converge to those from Black-Scholes-Merton closed form solution for a simple European style option [40], [41]. As mentioned before we treat each of the \( C^3 \) as assets and use the binomial lattice algorithm to price them.

4.2 Compounded Moore’s Law

This statement (law) by Gordon Moore [11] (that the number of transistors that can be placed on a circuit will double roughly every 18 months) has been holding true so far for processing power, memory etc. According to Vardi [Communications of the ACM, Vol. 57, No. 5, pg. 5, May 2014] current observations on technological developments seem to suggest that we are heading in to an uncharted territory. Moore’s law provides an estimate of the improvements in hardware design. However, to estimate the current price of Cloud infrastructure, factors such as rate of inflation should be considered. We use the compound interest formula along with Moore’s law for Cloud resource pricing. We call this Compound-Moore’s Law

\[
E_T = E_0 \times 2^{T/2},
\]

where \( E_0 \) and \( E_T \) are processing capacity of processors at time \( t = 0 \) and time \( t = T \), the maturity date of Cloud resource contract.

The future value of an asset \( S_T \) can be evaluated using the present value, \( S_0 \), the rate of interest \( r \) and the number of years \( n \) using the following formula:

\[
S_T = S_0 \times (1 + r)^n.
\]

The present value \( S_0 \) can be equated to the initial investment by the provider in building a Cloud data center and the future value is the initial investment’s worth at the end of the contract period. To price \( C^3 \), we consider the
depreciation of the existing infrastructure, inflation, and the technological evolution based on Moore’s law.

Combining Equations (2) and (3) we get the following equation for Compound-Moore’s law for any of the resource in the Cloud:

\[ X_T = X_0 \times (1 + r)^{T/2}, \]  

where \( X_0 \) and \( X_T \) are values of any of the resources in the Cloud at the initial time and at maturity respectively. This equation calculates the depreciation of resource \( X \) based on Moore’s law. However, in conjunction with compound interest formula presented above, the value of the resource \( X \) is computed indirectly through this equation as well. That is, Equation (4) now includes two very important aspects of a resource and its utilization, technological and financial, that are essential for appropriate and accurate resource pricing.

4.3 Pricing Algorithm

We identify the Cloud parameters and map them to financial option parameters in order to develop a resource pricing algorithms \{1\} and \{2\}.

4.3.1 Cloud Parameters

There are five parameters pertinent to pricing Cloud resources:

1. **Capital Investment** \((I_C)\). This gives the Cloud service provider’s expenditure per year. For example, a service provider might buy a resource \( X \) each year. According to the Compound-Moore’s law, for a given investment duration, the provider will reap more processing power at a constant price. Also, the service provider will pay less amount to buy the same resource \( X \) next year and even lesser in the subsequent years. When pricing the resources from the Clients’ perspective this is the estimated initial investment that the client would incur to install and own such a resource.

2. **Contract time** \((T)\). The time period the client wants to lease the resources from the Cloud service provider. From the client’s perspective, this could relate to the actual use-time of the resources for pricing.

3. **Rate of depreciation** \((\theta)\). It is the rate at which the infrastructure of service provider is expected to lose its value both financial and technological. The pricing policies of service provider should be such that they make profits on their initial investments before the clients no longer want to lease these resources. This information generally may not be available to the clients and hence while pricing from the client’s perspective this parameter is an estimate.

4. **Quality of service** \((QoS)\) \((r_s)\). This is the quality assurance from service provider to the client. This could include the turnaround time, accuracy of results, data privacy and contingency plans etc. QoS is the primary criterion while pricing the resources for services from both the provider and clients perspective.

5. **Age of resources** \((T_{res})\). It represents the age of a particular resource the service provider is leasing to the client. The start time of a particular task in a resource in conjunction with the age of the resource affect the price for the services.

4.3.2 Mapping Cloud Parameters to Binomial Lattice

The Cloud parameters discussed above can be mapped to five important parameters of the binomial lattice algorithm, \( S, K, r, t \) and \( \sigma \) as given below.

a) Quality of service \((r_s)\) is mapped to interest rate \( r \).

b) The **Total investment** \((I_{C_{total}})\) by the service provider is mapped to the asset price \( S \). It is the total amount that the service provider will spend during the lifetime of a contract and its value can be computed using Compound-Moore’s Equation (1) with Initial investment \((I_{C_{initial}})\) as one of the input parameter.

c) **Strike estimate** \((K_{est})\) is the equivalent of the strike price \( K \) of binomial lattice model, evaluated using Compound-Moore’s formula with Contract time \((T_t)\); and

d) **Volatility estimate** \((\sigma_{est})\) is the equivalent of the volatility of the underlying asset \( \sigma \) in the binomial lattice model with **Age of resources** \((T_{res})\) and **Rate of depreciation** \((\theta)\) as input parameters.

Algorithm 1. Pricing Cloud Resources

Get the input Cloud parameters

\[ I_{C_{total}} = \text{Compound-Moore}(T, I_{C_{initial}}). \]
\[ K_{est} = \text{Compound-Moore}(T, \theta) \]
\[ \sigma_{est} = \text{Compound-Moore}(T_{res}, \theta) \]

Map Cloud parameters to option parameters

\[ S \leftarrow I_{C_{total}} \]
\[ K \leftarrow K_{est} \]
\[ r \leftarrow r_s \]
\[ t \leftarrow T \]
\[ \sigma \leftarrow \sigma_{est} \]

Use Equation (1) to price the Cloud resource.

5 Value-at-Risk Analysis

It is in providers best interest to accommodate changes to the SLA to facilitate many clients. Clients, on the other hand, also acknowledge the fact that added requirements translate to a higher price. Therefore, it is the clients best interest to understand their business requirements before they request services from providers. As the SLA becomes more stringent, the vendor has to quote its prices to encompass all real and notional costs. The real cost includes the electricity charge, cost associated with high skilled personnel and software license fees etc. In general, the real cost is dynamic but identifiable to a large extent and hence can be quantified. Its the notional costs that are hard to evaluate. The notional expenses could include damages caused by fire or natural calamities and the expenses incurred due to SLA violations. In general, the notional expenses can originate from two sources of risk:

1. **Risk associated with vendor operations.** This is often referred to as the operational risks and are similar to any other kind of manufacturing units. For this study, the
operational risks are assumed to be very less and are not discussed further.

2. **Risk associated with SLA liabilities.** This risk is associated with the SLA violations. SLA violations can have different consequences on the Cloud providers; from a client not buying any more services to some more serious consequences involving litigations and penalties. When a Cloud provider is unable to provide the promised services, it results in a SLA breach ($P_{SB}$). All the possible clauses of a SLA breach are also a part of SLA. Cloud providers are very keen to eliminate the SLA liability risk by investing heavily in reliable hardware and software. However, it is very difficult to eliminate all the risks. All checks and cautions can reduce the risk to bare minimum still leaving a small opportunity for $P_{SB}$. A SLA breach will have some financial consequences associated with it, so the price quote of a Cloud provider should be adjusted to take into account this inherent risk. VaR estimator is a part of Clabacus and it is used to risk-adjust the price quote. The primary task of VaR estimator is to evaluate the probability of default or breach of a SLA. This probability, represented as percentage, would be added to the initially computed prices ($C_{base}$) to give a risk-adjusted price

$$C_{risk-adjusted} = C_{base} \times (1 + P_{SB}).$$

$P_{SB}$ can be evaluated using the QoS. Also, $P_{SB}$ is equivalent to the confidence level. It is the provider’s confidence that the SLA will not be breached. Confidence level at the base price gives us the VaR. Therefore, the equation can be rewritten as

$$C_{risk-adjusted} = C_{base} + VaR.$$

It can be seen that with higher VaR, the risk adjusted price of a Cloud resource will increase.

In algorithm \{3\} $\mu$ and $\sigma$ are the mean and standard deviation of the price $S$. This algorithm provides a generic way to evaluate VaR. The input parameters are price, mean, standard deviation and QoS.

The change in the price ($\Delta S$) can be computed using any of the three approaches described below: Monte-Carlo, fuzzy logic or genetic algorithm. Note that the fuzzy logic approach assumes static market conditions and hence $\Delta S$ need not be computed explicitly.

### 5.1 Monte-Carlo Simulation

Once the different values for $\Delta S$ are evaluated using MC, they are sorted in ascending order and a particular value of $\Delta S$ is selected based on the required confidence level, the VaR. Two limitations of MC are: (1) large number of simulations are required to get a reasonable value for VaR; (2) the MC assumes normal distribution of the price data, which does not reflect real market scenario. Due to these limitations with MC, in the current study, we compute the VaR using two different techniques: fuzzy logic and genetic algorithm.

### 5.2 Fuzzy Logic Approach

Fuzzy logic can be used when the boundary conditions are static, i.e., the market conditions are assumed to be constant.

In this section we demonstrate the use of fuzzy logic to evaluate the risk adjusted price using QoS ($r_q$) as an input parameter (Table 1). The QoS is directly proportional to the Cloud resource price. It should be noted that the $r_q$ would be derived from SLA.

We use fuzzy logic to determine the probability of breach from the QoS. Following is an example instance of boundary conditions in our fuzzification process.

Next step in the fuzzification step is to assign discrete and finite values to the variables separated by the boundary conditions. This assignment of values to each chunk is done by the use of common knowledge or historical data. We propose that Low $\mathcal{L}$ be associated with lower $P_{SB}$ and High $\mathcal{L}$ be associated with higher $P_{SB}$. With this fundamental heuristics, the $P_{SB}$ function can be derived as

$$P_{SB} = \left\{ \begin{array}{ll}
(92 - r_q)/6 & \text{if } \mathcal{L} \text{ is low} \\
(95 - r_q)/3 & \text{if } \mathcal{L} \text{ is medium} \\
(100 - r_q)/2 & \text{if } \mathcal{L} \text{ is high}. 
\end{array} \right.$$  

The above-mentioned assignments are for heuristic observations only.

A simple illustration of using Eq. (7) is shown below in Table 2.

In this section we presented heuristics to use the fuzzy logic to include the inherent risks associated with the Cloud providers. Risk-adjusted prices can help providers to maintain their profitability amid SLA liabilities. As clear from the above discussions, fuzzy logic relies on thresholds, which need to be set in order to estimate the VaR. These thresholds are static and need to be re-calibrated to capture real market scenarios, which is not easy and hence other simulation-based methods should be used to better incorporate VaR into Cloud resource pricing.

### 5.3 Genetic Algorithm Approach

We use Genetic algorithm to evaluate $\Delta S$. It should be emphasized again that GA is used to evaluate different values of $\Delta S$ only and not the actual VaR.

#### 5.3.1 Implementation Details of GA

A chromosome of length 4 can evaluate four different values of $\Delta S$. Consider two chromosomes of length 4 each. Each bit of a chromosome evaluates $\Delta S$ and in the fitness

<table>
<thead>
<tr>
<th>$C_{base}$ (cents/hours)</th>
<th>QoS (%)</th>
<th>$C_{risk-adjusted}$</th>
</tr>
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<tbody>
<tr>
<td>10</td>
<td>91</td>
<td>10.016</td>
</tr>
<tr>
<td>10</td>
<td>94</td>
<td>10.033</td>
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<tr>
<td>10</td>
<td>96</td>
<td>10.2</td>
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evaluation, we average all four bits to find the overall fitness of a chromosome.

1) **Initialization.** In this step we initialize all the bits with random initial prices and at the end of this step, chromosomes look like the values in Table 3.

2) **Crossover.** In this step the two initial chromosomes (parents) are recombined to form the next generation (Children) of chromosomes as presented in Table 3.

3) **Mutation.** Mutation can take place randomly at any bit and after mutation the chromosomes may look like the one in Table 3.

4) **Evaluate.** At this point all the bits in one chromosome are averaged to get the overall fitness. This overall fitness will dictate the selection of a particular chromosome for next generation.

Once the desired number of simulations is completed, the role of genetic algorithm is completed. At this time all the chromosomes are concatenated to get all the values at the end of last simulation. These values are now sorted and a particular value is selected based on the desired confidence level, which is the VaR. For example, the 95th lowest value selected from the sorted ΔS would mean 95 percent confidence level.

6 **EXPERIMENTS AND RESULTS**

We organize this section in the following order: we discuss the Clabacus input parameters first followed by the bounds on the Cloud resource prices that are beneficial for both clients and providers. Then, we discuss the effect of each of the input parameters on the Cloud resource pricing from the client and provider’s perspective separately.

### 6.1 Clabacus Input Parameters

The input parameters to Clabacus are as follows:

- **Capital investment (I_c):** this is the approximate cost of the new equipment or infrastructure the client wants to lease from a Cloud resource provider.

- **Start time:** this is the time when the client starts leasing the Cloud resources.

- **Use time:** this is the duration for which the client actually uses the resources.

- **Total time:** is the time for which the Cloud provider will possess a resource. For example, a total time of 2 years means that after 2 years the Cloud provider will dispose the resource from memory.

- **Rate of depreciation(θ):** is the rate at which the Cloud resource depreciates. This depreciation could be the result of several factors such as the advent of new technology or increased cost of maintenance. One direct effect of this parameter can be seen in the unwillingness of clients to lease a particular (aged) resource.

- **Quality of service:** is the conglomeration of many factors including completion time, accuracy of results, data confidentiality etc.

- **Rate of inflation:** is the rate of change of prices on an annual basis.

### 6.2 Lower and Upper Bounds on Resource Prices

This section explains the maximum and minimum cost the service provider would charge a client in leasing a resource.

#### 6.2.1 Upper Bound from Compound-Moore’s Law

The five Cloud parameters are: capital investment, contract time, rate of depreciation, quality of service and age of resource. Using these parameters together with compound-Moore’s law, we can compute the maximum amount the service provider is spending in buying and setting up the hardware resources. In other words, this provides an upper bound on the Cloud resource price, the service provider would like to charge a client to recover the investment over the contract period.

The cost of maintenance (including power, real estate and personnel etc.) is not considered in the computation block for two reasons:

- a) Our objective is to make Clabacus usable by both users and providers.

- b) The revenue generated by providing service to multi-tenants (at a low cost to them) from a given virtualized resource can compensate the maintenance cost to the provider.

Please note that this limitation is relaxed in the Value-at-Risk analysis which takes into account all the recurring costs. Also, risks due to rate of depreciation, rate of inflation, etc. are taken into consideration to adjust the resource prices computed in the computational block. That is, we can expect that adding the cost of maintenance in the price evaluation would bring profits to the provider at an earlier time.

#### 6.2.2 Lower Bound from Finance Models

Using the binomial lattice algorithm or any other techniques (numerical or heuristics) we can compute the minimum Cloud resource price, the service provider would charge a client to recover its initial and recurring investments. In other words, this is the lower bound on the Cloud resource price.

#### 6.2.3 Example and Observations

We explain the upper and lower bound concepts mentioned above through an example. A typical parameter setting for our experiments is: Capital Investment: $300/year; Contract (use) period: 3 years; Rate of depreciation: 10 percent; Quality of Service: 0.4; Age of resources (total time): 2 years.

Using compound-Moore’s law, the upper bound can be calculated as 2.14 cents/hour. This is the maximum amount the service provider would need to charge a client to recover the investment over the contract period. The lower bound using the binomial lattice algorithm is calculated as 1.65 cents/hour. This implies that the service provider would
need to charge a client at least 1.65 cents/hour but not more than 2.14 cents/hour to recover the initial investment.

With the proposed methodology the client is aware of the maximum cost of leasing the resources from the service provider which is beneficial for the client to compare this cost with respect to buying the resources. In this regard the client would benefit a low cost resource if the price of the leased resource is less than 2.14 cents/hour. Hence, we are reaching an equilibrium condition and since the prices are adjusted continuously for the fluctuation in the market conditions, the resource prices are in dynamic equilibrium.

This is just a break-even point where the clients and vendors share the profits equally. Proximity of the actual price charged by a vendor would depend on other factors like market competition. In an ideal rational market, the vendor will always charge this equilibrium price. The price that the vendor would charge a client should ideally be between these bounds. When the price is between lower bound and equilibrium the vendor would make better profit while the client will be benefitting by leasing the resources. When the price is between equilibrium and upper bound, vendor would make better profits while at the same time the client will benefit based on how close to the bound the price is.

Note that the input parameters mentioned earlier in this section are investments at the time of hardware installation. These parameters, especially the capital investment, need to be adjusted for decreasing prices due to technological evolution as mentioned in Section 4.3.1. We do this using compound-Moore’s law before applying the parameters to Algorithm 2 in computing the upper bound. Also, note that the cost to service provider depends on the initial capital investment and remains at 2.14 cents/hour, the upper bound. However, at 100 percent quality of service (i.e., QoS = 1) cost to client is computed as 2.06 cents/hour. That is, the lower bound is 2.06 cents/hour.

Algorithm 2. Compound-Moore (T,a)

\[
X = a \times (1 + r_a)^{T/2};
\]

return \(X\);

Algorithm 3. Risk-Adjusted Price for Cloud Resources

Get prices from the pricing algorithm \(1\)
For \(i = 1, N\) (\(N\) is the number of iterations)
Evaluate \(\Delta \bar{S} = f(S, \mu, \sigma)\)
\[C_{\text{risk-adjusted}} = C_{\text{base-price}} \times (1 + \mathcal{P}_{SB})\]

In general, if the client had to purchase the resource, the value of the client’s infrastructure investment at risk is 100 percent. However, by leasing the resources from a Cloud service provider the client has none of the investment at risk of losing value. That is, the value-at-risk for clients is significantly lower.

The example explained above involves one client and one service provider; however in reality a single service provider will have many clients. With virtualization of the resources, the service provider can cater to the needs of many clients from a given physical resource, thus generating more revenue on the initial investment and hence make profit.

Based on these observations it can be understood that client and the service provider can have a symbiotic relationship. Experiments results presented in the next sections show the influence of various Cloud parameters on the Cloud resources.

6.2.4 Risk-Adjusted Pricing

We modify the prices computed using any of the algorithms in the computational block to reflect the risks involved (as described in Section 5) in providing Cloud services. We present the results from our proposed genetic algorithm.

6.3 Discussion: Pricing from Client’s Perspective

In an experiment when studying the effect of one parameter on the resource price, other parametric values are kept constant at a desirable level. Also, note that the experimental results in the figures are for a slightly different parameter setting than in Section 6.2.3.

6.3.1 Effect of Start Time on Resource Price

Start time is the time of the client’s first instance of using the Cloud resource. In Fig. 2, we delay the start time to see the effect on resource pricing. Over a period of time the resources the clients has acquired from the Cloud provider ages, thereby decreasing the value of the resources. Here, the rate of depreciation is the implicit factor affecting the resource price. This also affects the quality of service obtained from Cloud provider. In other words, using the compound-Moore’s law, we show that a client can estimate the rate of depreciation and the effect it has on resource pricing as shown in Fig. 2. However, we may expect that when the pricing is done by the provider, the age of the resource should not show any effect on the resource price, as long as the quality of service is maintained. This is realized as presented later in Fig. 12.

6.3.2 Effect of Use Time on Resource Price

The use time in Fig. 3 is the duration of leasing the resources. Trend in this figure is not only the reflection of expected price rise as use time increases, rather this is counter intuitive that longer a client uses a resource the client would expect to pay lesser. Higher use time on a resource may be interpreted as higher demand on the resource. Note that as
the demand of resource increases, the provider will increase the price of the resources. This has to be taken into account by the client. This is not easily predictable since the provider may not divulge the information of the load of the resources to the client. However, the computational module in our model has the ability to predict the resource prices over a period of time. For example, in the current study, we estimate or predict future prices by emulating many price evolution in the binomial lattice algorithm for a Cloud resource. The resource price computed from binomial lattice is a result of many possible price evolution in the future. In other words, many possible variation in prices are evolved by regenerating the binomial tree for various $u$, $d$ and $\sigma$ values to create a totally different market condition and hence appropriate prices using equation (1). Incorporating this financial option based binomial-lattice algorithm (and others in the computational module) allows us to better estimate the price of the resources also, which would not have been possible with simple economic models. This is shown in Fig. 3.

6.3.3 Effect of Rate of Depreciation on Resource Price

It is expected that the price would drop at higher depreciation level of the resources (Fig. 4). This figure is related to Fig. 2 intuitively. The rate of depreciation is an important factor when leasing a Cloud resource. Note that in algorithm (1) the rate of depreciation is an input parameter to the compound-Moore’s law formula. By finding the strike estimate (which is the resource price) using the binomial lattice algorithm, our model can predict the rate of depreciation for the client as shown in Fig. 4. With a depreciating resource the provider has to incur maintenance costs without which the service charges would decrease as shown in Fig. 4. Note that in this experiment, other parameters are set to an initial condition, especially the QoS, which cannot be maintained without proper maintenance.

6.3.4 Effect of Quality of Service on Resource Price

With the increased QoS it is expected that demand on a resource would increase, thereby, increasing the price of the resource. The binomial lattice model together with the compound-Moore’s law captures the resource price (upper bound) for increasing QoS as shown in Fig. 5. In this figure, our model provides the upper bound price for higher QoS. We can obtain this using our algorithm (1) where “$r$” in the binomial lattice algorithm represents QoS. By mapping the appropriate parameters to the Equation (1), we compute the price of the resources with respect to the QoS.

6.3.5 Effect of Rate of Inflation on Resource Price

The increased running and maintenance cost can increase inflation rates. We can predict the cost of burden on the client due to inflation using our model. Here, we estimate the volatility using the age of resources and rate of depreciation using compound-Moore’s law. Also, we calculate the strike estimate using rate of depreciation and contract time or total time. We map these parameters to the binomial lattice algorithm to obtain the increase in resource prices due to inflation as seen in Fig. 6. This means that higher prices is the result of any of the causes such as electricity bill, taxes etc. to the provider.

6.3.6 Effect of Capital Investment on Resource Price

If a client wants state-of-the-art machine (requiring higher capital investment), the resource price increases proportionally for increasing capital investment as can be seen in Fig. 7. Capital investment refers to the approximate cost of the Cloud infrastructure that a client would like to acquire instead of leasing from a Cloud resource provider. Given a contract time, initial investment, and the use time of the resources, we can calculate the total investment, strike and
volatility from algorithm \([1]\) using the rate of depreciation and age of resources. Our model uses these parameters to provide the cost of leasing this infrastructure.

6.4 Discussion: Pricing with Provider’s Perspective

6.4.1 Effect of Capital Investment on Resource Price
The effect of increasing capital investment on Cloud resource can be seen in Fig. 8. We see that the resource price (asking price) increase is proportional to the initial investment of the service provider. This proportionality is due to the fact that the contract period is kept constant. Our algorithm allows us to vary the contract time between a single client and provider. Handling multiple clients with varying contract periods becomes a problem of resource allocation first. Once the tasks are assigned to the appropriate resources, we can price the resources. However, we have not considered the task assignment problem in this study.

6.4.2 Effect of Contract Time on Resource Price
The effect of contract time on the Cloud resource price can be seen in Fig. 9.

It can be seen that it is beneficial for a client to lease the Cloud resource for a longer time; the prices decrease as the contract time increases. That is, longer contract periods could benefit from the use of Cloud resources to a larger extent than the smaller contracts. This is due to two reasons: (1) the resource price variation could average out over a long period of time; note that when compared to shorter contracts (minutes) the 2 year contract that we have in our parameter setting can be considered very long period, during which time the price variation is assumed to be normally distributed. and (2) smaller jobs may get executed at a time when the resource price is at its peak, which is still between 2.14 and 1.65 cents per hour. Note that in the current set up, as long as the price is between the lower and upper bounds, both Cloud provider and client are benefited.

6.4.3 Effect of Rate of Depreciation on Resource Price
The expected rate of depreciation of the hardware installed by the service provider is very critical to price the Cloud resource. As explained earlier, if the rate of depreciation is high the service provider would like to recover its investment before the hardware becomes obsolete, which in turn would increase the price of Cloud resource. This can be seen in Fig. 10. By cross referring this figure to Figs. 4 and 11, we can conclude that the Clabacus model brings equilibrium to the providers and clients implicitly.

6.4.4 Effect of Quality of Service on Resource Price
Higher the quality of service the client demands, more is the asking price from the service provider as evident from the

![Rate of inflation Vs Resource price](image1)

Fig. 6. Rate of inflation versus resource price.

![Capital Investment Vs Resource Price](image2)

Fig. 8. Effect of capital investment on the resource price.

![Capital investment Vs Resource price](image3)

Fig. 7. Capital investment versus resource price.

![Resource price Vs Contract time](image4)

Fig. 9. Effect of contract period on the resource price.
Fig. 11. A price range (lower and upper bounds) presented earlier is still valid for this discussion. That is, the upper bound price would correspond to the highest QoS. In other words, the compound-Moore’s law based pricing and binomial lattice model based pricing form boundaries of the price range for which the QoS varies proportionately. This is consistent with the pricing from the clients perspective as presented in Fig. 5.

6.4.5 Effect of Age of Resource on Resource Price
The age of resource had no impact on the Cloud resource price as shown in the Fig. 12. This is because the quality of service and the rate of depreciation are kept constant as we varied the age of resources in our simulations. This implies that the provider is concerned about the quality of service rather than the hardware used to accomplish the task and not to breach the SLA with the set price at the time of contract in completing the task. However, the Cloud service provider might incur more expenses managing aged resources. The client is completely immune to it.

6.5 Equilibrium Pricing
Analyzing the blended effect of the parameters on the resource pricing is a natural next step and this is a multi-objective price optimization problem. Though we have not studied this extensively, one simple example is presented in Fig. 13, where the resource is priced from both the client and provider perspective for one set of parametric condition different from earlier sections due to blending. An equilibrium or break-even price of 0.58 cents/hour is obtained at 0.425 years of contract. This result helps a client to make a business decision between buying and leasing. It is beneficial for the client to lease the resource if period of use is below 0.425 years; in other words, the client would exercise the option contract. If the client is going to use the resource for longer than 0.425 years s/he need not exercise the option, that is, he need not lease the resources from the provider and instead it would be better to invest in buying the infrastructure. In financial markets, an investor can estimate the premium they expect to pay by computing the option value using any of the pricing models. Recall that the holder of a financial option has the right to exercise his/her right. The Fig. 13 signifies this for the client who is the holder of the contract. Clabacus enables a client to make such a business decision to either buy or lease a resource by knowing an equilibrium price.

Fig. 10. Effect of rate of depreciation on the resource price.

Fig. 11. Effect of rate of depreciation on the resource price.

Fig. 12. Effect of age of the resource on the resource price.

Fig. 13. Equilibrium price.
From this figure, obviously it is not beneficial for the provider to run a data center for one client with a contract time larger than 0.425 years. However, knowing the overall cost of running the data center and using the data from this figure the provider can compute the minimum number of clients required to reach a break-even point. In other words, for the Cloud provider the financial impact would be influenced by the number of clients it has among other factors at any given time. The minimum required number of clients can be found using this figure. Referring back to the option pricing models, the writer of an option quotes the premium cost to sell the option based on the market competitors. Similarly, the Cloud provider would base his pricing decisions on the market factors and not only on the equilibrium price computed using the option-based models.

7 Conclusions and Future Directions

We presented a quantitative approach to price Cloud resources from both client and provider’s perspective with our proposed Clabacus (Cloud Abacus) architecture. Clabacus has the capability to recognize the input parameters and map them to pricing models appropriately. The challenge in mapping Cloud computing parameters to financial option model(s) is resolved by carefully analyzing individual parameters and their effects. We treated the Cloud resources as assets in a finance model to capture the realistic value of the cloud compute commodities and used financial option theory concepts and algorithms to solve the mathematical model. The finance model provided a lower bound on the prices. The upper bound is found using our proposed compound-Moore’s law that takes into account various metrics (such as start time of the resource, use time, rate of depreciation, quality of service, rate of inflation, and capital investment). Risks faced by a provider is another major issue that affects the pricing of Cloud resources. We have addressed this challenge quantitatively through our proposed fuzzy logic and genetic algorithm based approaches to find the investment value-at-risk. The resource price computed in our computation module is adjusted with the VaR to arrive at the final resource price, and showed that the final price computed is still between the lower and upper bounds making the final price competitive to clients and profitable to the provider.

The blended effect of the parameters on the resource pricing is a natural next step and this is a multi-objective price optimization problem. One such simple example was presented in Fig. 13 using financial option pricing model. Nash equilibrium economic principle could be used to achieve such a result, which is an interesting direction to pursue for further research.

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