Inverse Queuing Model-Based Feedback Control for Elastic Container Provisioning of Web Systems in Kubernetes

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Abstract—Container orchestration platforms such as Kubernetes and Kubernetes-derived KubeEdge (called Kubernetes-based systems collectively) have been gradually used to conduct unified management of Cloud, Fog, and Edge resources. Container provisioning algorithms are crucial to guaranteeing quality of services (QoS) of such Kubernetes-based systems. However, most existing algorithms focus on placement and migration of fixed number of containers without considering elastic provisioning of containers. Meanwhile, widely used linear-performance-model-based feedback control or fixed-processing-rate-based queuing model on diverse platforms cannot describe the performance of containerized Web systems accurately. Furthermore, a fixed reference point used by existing methods is likely to generate inaccurate output errors incurring great fluctuations encountered with large arrival-rate changes. In this article, a feedback control method is designed based on a combination of varying-processing-rate queuing model and linear-model to provision containers elastically which improves the accuracy of output errors by learning reference models for different arrival rates automatically and mapping output errors from reference models to the queuing model. Our approach is compared with several state-of-art algorithms on a real Kubernetes cluster. Experimental results illustrate that our approach obtains the lowest percentage of service level agreement (SLA) violation (decreasing no less than 8.44 percent) and the second lowest cost.

Index Terms—Cloud, fog and edge computing, containers, container auto-scaling, QoS control, queuing theory, feedback control

1 INTRODUCTION

One of the most effective approaches to share resources of diverse systems such as private data centers, Cloud Computing, Fog and Edge Computing [1] among multiple applications is using containers which are more lightweight and portable than virtual machines (VMs) [2]. Kubernetes [3] is one of the popular container orchestrating systems and is gradually used to manage Cloud, Fog and Edge resources by containers seamlessly leading to many derivative platforms such as KubeEdge [4]. Kubernetes’s Pods (consisting of one or more containers) of different applications are deployed on VMs rented from public Clouds or physical machines (PMs) of Fog and Edge nodes. Meanwhile, Web applications and services (called Web systems collectively) are very common in Cloud, Fog and Edge Computing providing various functions to end users via different micro-services deployed in the form of containers. One of the most crucial problems is to design container auto-scaling algorithms to control response time of each micro-service in Kubernetes-based platforms.

Most existing container provisioning algorithms focus on placement and migration of fixed number of containers on PMs or VMs rather than auto-scaling of containers [5], [6], [7]. However, the number of containers allocated to each micro-service has a great impact on request response times. Kubernetes’s built-in auto-scaling scheduler [8] and most threshold-based methods [9] only add or remove container replicas based on resource usage rates which are indirect metrics for controlling response times. Therefore, the main goal of this paper is to design container auto-scaling methods for Kubernetes which adjust the number of containers allocated to each micro-service automatically to decrease resource consumption while guaranteeing quality of services (QoS). The main challenges of designing such auto-scaling algorithms include non-linear performance model of multi-container systems and finding appropriate output error computing methods.

Non-linear performance characteristics of Web systems make resource auto-scaling complex. QoS control has been studied extensively for traditional Web systems involving elastic provisioning of application resources, PMs or VMs. However, most existing methods belong to pure queuing-theory-based feed-forward control [10], [11], [12], [13] or linear-model-based feedback control [14], [15], [16] which lack feedback ability or cannot describe complex non-linear multi-container systems accurately. Although linear-model-based feedback control has been used to amend the inaccuracy of queuing models taking advantage of queuing and control theory together, the reference-point-derived linear...
performance model only works well when the system is near the reference point [17], [18].

The deviation between the reference point and the real-time response time is called output error which has a great influence on control performance. Selecting appropriate reference points to compute output errors is helpful to obtaining a stable control performance. In existing methods [19], reference points are usually selected manually and kept unchanged for different arrival rates. However, a fixed reference point is likely to incur great control fluctuations because there are different stable working points for different arrival rates. Meanwhile, the inconsistency between the profiled performance model and the real system is unavoidable which makes output errors mismatch with the profiled performance model.

In this paper, an inverse-queuing-model-based feedback control method (FeedBack_InverseQM) is proposed to guarantee the QoS of container-based Web systems in Kubernetes which decreases the percentage of SLA violation by 8.44 percent. The main contributions of our work are as follows.

1) A hybrid of varying-processing-rate-based queuing model and linear model is applied to describe the performance of the multi-container system more accurately. Inverse-queuing model is used to linearize the control system to simplify the controller design.

2) An online reference model learning method is designed to find appropriate reference points for different arrival rates increasing the accuracy of output errors.

3) An adaptive output-error-mapping method is proposed to amend the inconsistency between sampled reference models and the profiled performance model avoiding fierce control fluctuations.

The rest of this paper is organized as follows. Section 2 is the related work and Section 3 describes Web systems in Kubernetes. The proposed method is introduced in Section 4. Sections 5 and 6 include performance evaluation on a real Kubernetes cluster, conclusions and future work.

## 2 RELATED WORK

Most of existing works for container scheduling assume that containers consumed by each application is fixed. Deploying and migrating a fixed number of containers on private or elastically rented underlying resources can be modeled as a bin-packing problem [7] and solved by CPLEX [6]. For Kubernetes, heuristic methods such as best fit decreasing bin packing (BFD) and time-bin BFD [5], [20] have been proposed for the container deploying and migrating. Reinforcement learning [7] and game theory [21] are also used to deploy and migrate a fixed number of containers considering geography locations of Fog nodes, user mobility, energy consumption or SLA violations.

Compared with assigning fixed number of containers, providing scalable computing and storage capacities to applications is beneficial to saving resource cost and improving efficiency. For example, the authors of [22] propose a scheme to support data auditing processing in Cloud with fine-grained data updates (i.e., flexible size data updates). It saves huge storage and computation overheads of data auditing processing against fixed-size update where every small update will cause re-computation and updating for an entire file block. This scheme offers highly scalable and efficient data auditing processing in Cloud. To provide scalable computing capacity to applications in Kubernetes, genetic algorithms [23] and linear programming [24] have been used to find the optimal container distributing policy among multiple applications assuming that each application has a performance-degrading resource threshold or the utility of provisioning different numbers of containers to each application has been profiled in advance. In other words, these works mainly focus on the distribution of resources among competitive applications while container auto-scaling of one application is considered in this paper. Since auto-scaling methods of traditional application resources, PMs and VMs can be migrated to deal with container auto-scaling, auto-scaling techniques are surveyed in total.

### 2.1 Auto-Scaling Techniques for QoS Control

Threshold is one of the most simplest auto-scaling technique. Kubernetes’s build-in auto-scaling scheduler adjusts container numbers based on thresholds of CPU or Memory usage rates [8]. One container is added when existing resources have been used up [9] or a fixed number of containers are added when the real-time response time is larger than a threshold [25]. The difficulty of threshold-based methods is how to select appropriate threshold values suitable for different arrival rates [26].

Queuing models describe the relations among the average response time, the request arrival rate and allocated resources of Web systems. M/M/1 [12], M/M/N [10], [11], heterogeneous M/M/N [27] queuing models and queuing networks [13] have been used to determine the minimum number of VMs or PMs for guaranteeing QoS of Web systems. Nonetheless, pure queuing-model-based methods lack the ability of reacting to real-time output errors.

Feedback control is an essential method for application resource (server processes, database connections, etc) auto-scaling. Linear-performance-model-based fixed gain [14], [15], adaptive [16] or multi-model switching [28] feedback control methods have been widely used in application resource auto-scaling [29]. An inverse-proportional performance model, which is more accurate than linear models, is used to design a feedback control method to auto-scale containers of Web applications [19]. Queuing models are more accurate than linear or inverse-proportional models, and have been used to improve the performance of feedback control. For example, M/M/1 queuing model-derived linear model, which describes the linear relation of output-error changes and the adjustment of allocated resources near the reference point, is used to design feedback controllers, and only works well near reference points [17], [18]. For Kubernetes, M/M/1-model-based feedback controller is designed to adjust the arrival-rate adjustment coefficient of a loosely coupled M/M/N model [30] which is tailored for avoiding over-control incurred by interval-based charging models of Cloud VMs.

Q-table and deep-neural-network-based reinforcement learning methods have been used to allocate VMs or PMs to Web systems elastically [12], [31], [32]. However, these
methods need a long sampling and training period to obtain good performance [33]. Therefore, exact model based methods are still necessary and can be used to guide the deep learning based methods.

2.2 Comparison With Existing Algorithms

Table 1 shows a comparison of our approach with existing resource provisioning algorithms. First, existing linear or queuing-model-derived linear models cannot describe the performance of container based systems accurately [34]. On the contrary, our approach applies a feedback controller based on a more accurate performance model which is the hybrid of varying-processing-rate $M/M/N$ model and linear model. Second, most existing works use a fixed reference point obtained by experience to compute output errors directly, while our approach applies an automatic reference point identification method and an adaptive output error mapping method to improve the control stability. Finally, our approach is implemented as a user-level scheduler of Kubernetes rather than simulation platforms.

### 3 Web Systems in Kubernetes

It is flexible and inter-operable to organize diverse Web systems using micro-service-based architecture. Each tier of such Web systems can be implemented as a micro-service which runs in parallel containers to support large-scale requests. Each micro-service provides a single function and is usually encapsulated as a RESTful or SOAP-based Web service. Kubernetes is widely used to manage containers which includes one Master and multiple Worker nodes as shown in Fig. 1. Pod is the basic unit of resource management which consists of one or multiple containers [3]. Containers of micro-services are embedded in Pods which are deployed on Worker nodes (PMs or VMs). Containers of the same color belong to the same micro-services. User requests of the same micro-service are distributed to parallel containers by the nginx-ingress-controller [36]. Real-time performance of each micro-service is monitored through Kubernetes java-client-interface [37], based on which the

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<td>Linear-model-based feedback control, $O(1)$ Queuing-model-derived linear model based hybrid control, $O(n^2)$</td>
<td>A single Web Server</td>
<td>[14], [15], [16], [28]</td>
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<td>Auto-scaling</td>
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<td>Threshold, $O(1)$ Reinforcement learning Queuing models, $O(n^2)$ Queuing-length-based feedback control, $O(1)$ Queuing-model-arrival-rate-adjusting-coefficient based hybrid control, $O(n^2)$</td>
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<td>Kubernetes, Simulation</td>
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<td></td>
<td>Resource usage, Average response time, Total network latency, etc Average response time</td>
<td>Threshold, Genetic algorithm Simple non-linear model based feedback control, $O(1)$ Inverse-$M/M/N$-model based feedback control, $O(n^2)$</td>
<td>Kubernetes</td>
<td>[19]</td>
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<td>[19]</td>
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Fig. 1. Architecture of container-based web systems in Kubernetes.
container Auto-scaling Controller (ASC) is implemented as a user-level plugin to allocate appropriate number of containers to each micro-service separately. Each micro-service has its own ASC, i.e., a decentralized controlling method is applied [19].

The average response time and resource cost are two crucial metrics of containerized Web systems. In the SLA of a micro-service, it is usually defined that $x\%$ of response times should be smaller than a given threshold $W_{sla}$. Because identical containers are usually required by the same micro-service, one Container Unit (CU) is defined to be the cost of allocating one container to a micro-service in one control interval. The objective of this article is to design container auto-scaling algorithms for ASCs to decrease the number of consumed CUs while guaranteeing SLAs. Common notations of this article are shown in Table 2.

### 4 PROPOSED FEEDBACK CONTROL METHOD

In this paper, an Inverse-Queuing-Model-based feedback control method (Feedback_InverseQM) is proposed. A hybrid of varying-processing-rate-based M/M/N model and linear model is first adopted to describe the system accurately. Then an automatic reference model learning method is developed to generate accurate output errors. Based on the performance model and output errors, an inverse-queueing-model-based Integral controller is designed. Next an adaptive output error mapping method is investigated to amend the inconsistency between sampled reference models and the profiled queuing model. Finally, a queuing-length-based scheduling method is used to provision containers when the system is unstable.

#### 4.1 Varying-Processing-Rate-Based Performance Model

Performance model is the basis of feedback control. The average processing rate of one container decreases as the request arrival rate increases in Kubernetes because of scheduling overhead. Traditional queuing models with fixed processing rates-based performance models cannot describe the system accurately. Let $\lambda$ be the request arrival rate and $N$ be the number of containers. In this paper, the processing rate of each container is defined to be inversely-proportional to the arrival rate as

$$\mu = \mu_b + c/\lambda,$$

where $\mu_b$ is the basic processing rate and $c$ is the inverse-proportional coefficient of $\lambda$. According to existing M/M/N model [10], [11], the probability of no requests in the whole system is

$$P_0 = \left[ \sum_{k=0}^{N-1} \frac{1}{k!} \left( \frac{\lambda}{\mu} \right)^k \frac{\lambda^N}{N!(1 - \frac{\lambda}{\mu})} \right]^{-1}. \quad (2)$$

The expectation of the number of requests in the waiting queue and under processing is

$$L_s(N, \lambda, \mu) = \frac{\lambda^N}{N!(1 - \frac{\lambda}{\mu})^2} P_0 + \frac{\lambda}{\mu}. \quad (3)$$

The expectation of the average response time of requests is

$$W_s(N, \lambda, \mu) = \frac{L_s(N, \lambda, \mu)}{\lambda}. \quad (4)$$

Meanwhile, the current average response time is also affected by past values. Therefore, in this paper, the average response time $y_k$ of control step $k$ is defined to be a weighted combination of the past value $y_{k-1}$ and $W_s(N, \lambda, \mu)$ as

$$y_k = a \times y_{k-1} + (1 - a) \times W_s(N, \lambda, \mu). \quad (5)$$

Equation (5) is a hybrid of inverse-proportional-processing-rate (varying processing rate) based M/M/N queuing model and linear model which is applied to be the performance model of this paper.

#### 4.2 Automatic Reference-Model Learning

Reference points are crucial to compute output errors which are the basis of feedback control. Because containers are provisioned in a coarse-grained granularity, average response times (called stable points) cannot change smoothly when the number of provisioned containers is adjusted as shown in Fig. 2. Therefore, the given $W_{sla}$ might be far from any stable point, and using $W_{sla}$ as the reference point directly is likely to generate great fluctuations. Appropriate reference response times should be selected from stable points smaller than $W_{sla}$. Fig. 2 illustrates that stable points of different arrival rates are different, the reference point $W^r$ should be found for each arrival rate individually. For each $\lambda$, the maximum number of containers $n^r$ violating SLA is called the lower bound which is able to avoid releasing too many containers. For each $\lambda$, stable points of different numbers of containers can be studied from historical data.

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**TABLE 2**

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\lambda$</td>
<td>Real-time request arrival rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Varying processing rate</td>
</tr>
<tr>
<td>$W_{sla}$</td>
<td>Maximum response time described in SLA</td>
</tr>
<tr>
<td>$k$</td>
<td>Index of control steps</td>
</tr>
<tr>
<td>$y_k$</td>
<td>Real-time average response time of step $k$</td>
</tr>
<tr>
<td>$N_k$</td>
<td>Output container number of control step $k$</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Available containers for one microservice</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Real-time queuing length</td>
</tr>
<tr>
<td>$m_k$</td>
<td>Reference model of step $k$ with maximum arrival rate deviation of 5/s</td>
</tr>
<tr>
<td>$m'_k$</td>
<td>Mature reference model of step $k$ with maximum arrival rate deviation of 20/s</td>
</tr>
<tr>
<td>$W^r_r$</td>
<td>Reference response time of $m'_k$</td>
</tr>
<tr>
<td>$n^r_{m'_k}$</td>
<td>Maximum number of containers not fulfilling SLA in $m'_k$ (called lower bound)</td>
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</table>
$W^r$ and $n^r$ can be obtained base on stable points of each $\lambda$. The set of stable points, $W^r$ and $n^r$ is called a reference model of $\lambda$ which is used to guide the feedback control.

An automatic reference-model learning method (ARML) is proposed to profile reference models based on samples in the form of $s = (n_k, \lambda, y_k)$ in which $n_k$ is the container number, $\lambda$ is the arrival rate and $y_k$ is the average response time. $M$ is the set of reference models. In order to decrease the number of studied reference models, the difference between any two models’ reference arrival rates should be larger than a gap $g$ (e.g., 5/s). Samples with similar arrival rates are used to profile the same model.

Formal description of ARML is shown in Algorithm 1. In auto-scaling step $k$, a new sample $s$ will be collected and a corresponding reference model $m_k = \arg \min_{m \in M} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g \}$ is tried to be found. If $m_k = \text{null}$, a new model $m_k$ with reference arrival rate $\lambda$ is created and added to $M$. If $m_k = m_k-1$ which means that the system’s arrival rate is stable in two consecutive steps, $s$ will be processed as follows. If $y_s > W_{sla}$ and $n_s > n^r_{m_k}$, $n_s$ is a larger number which violates SLA and $n^r_{m_k}$ is replaced by $n_s$. For example, $n^r_{m_k} = 10$ means that providing 10 containers will lead to SLA violations, but allocating 11 containers is able to guarantee SLA. When a new sample is obtained with $n_k = 13$ and $y_s > W_{sla}$ it means that allocating 13 containers is already not able to fulfill SLA. Therefore, 13 is a new maximum number not fulfilling SLA. When $y_s \leq W_{sla}$, $s$ is stored in a hashmap $h_{m_k} = \langle n, b_{n} \rangle$ in which container number $n$ is the key and bucket $b_{n}$ contains average response times of samples with container number $n$. For each $n$, only latest ten average response times are stored in $b_{n}$ and values with distances larger than $0.4 \times \text{average}(b_{n})$ from the average value $\text{average}(b_{n})$ are filtered. Then $\text{average}(b_{n})$ is updated based on filtered values for each $n$ and added to a set $Y$. Finally, the third largest value in $Y$ is taken as the reference response time $W^r_{m_k}$. For example, in Fig. 2, the hollow circles are the third largest average response times when $W_{sla} = 0.1$ s. The reason of not selecting the first two largest stable points is that small workload fluctuations might lead to SLA violations because the first two largest stable points are near $W_{sla}$. If $n_s \leq n^r_{m_k}$ which means a smaller or equal number of containers already can fulfill SLA compared with $n^r_{m_k}$, $n^r_{m_k}$ is updated to be $n_s - 1$. Whenever a new sample $s$ is collected from the system, $W^r_{m_k}$ and $n^r_{m_k}$ are updated as mentioned above. Only when $n^r_{m_k}$ has been assigned a value and the number of hashmap’s keys ($h_{keys}$) is larger than four, the model $m_k$ is called mature and added to a mature model set $M'$. Because immature models do not have sufficient samples to provide appropriate reference points and lower bounds, they cannot be used.

After $s$ is added to $m_k$, a mature reference model $m^r_k = \arg \min_{m \in M'} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g \}$ is tried to be found. $g$ is usually set to be larger than $g$ (e.g., 20/s) to allow mature models to guide more scenarios with larger arrival-rate differences. $m_k = m_k$ only when $m_k$ is mature. If $m_k$ can be found, $W^r_{m_k}$ is selected as the reference response time which will be used as the reference point of feedback control. Otherwise, a sampling method is activated to collect more samples for $m_k$. Let $n^r_{m_k}$ and $n^r_{m_k}$ be the smallest and largest keys of $h_{m_k}$. If $n^r_{m_k} - 1 > n^r_{m_k}$ or $n^r_{m_k} = \text{null}$, container number $N_k$ of current step $k$ is set to be $n^r_{m_k} - 1$. Otherwise, $N_k = n^r_{m_k} + 1$.

### Algorithm 1. Automatic Reference Model Learning (ARML)

```
input: $s, \lambda, m_{k-1}, W_{sla}$
1 Initialize $g = 5/s, g = 20/s, Y = \emptyset$
2 $m_k \leftarrow \arg \min_{m \in M} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g \}$
3 if $m_k = \text{null}$ then
   4 Create $m_k, \lambda_m \leftarrow \lambda$ and $M \leftarrow M \cup \{ m_k \}$
5 if $m_k = m_{k-1}$ then
   6 if $y_s > W_{sla}$ and $(n_s > n^r_{m_k}$ or $n^r_{m_k} = \text{null})$ then
     7 $n^r_{m_k} \leftarrow n_s$
   8 if $y_s \leq W_{sla}$ then
     9 Store $s$ in hashmap $h_{m_k}$
   10 for each $n \in h_{keys}$ do
     11 Calculate average($b_{n}$)
     12 $Y \leftarrow Y \cup \{ \text{average}(b_{n}) \}$
     13 $W^r_{m_k} \leftarrow$ the third largest value in $Y$
     14 if $n_s \leq n^r_{m_k}$ then
       15 $n^r_{m_k} \leftarrow n_s - 1$
     16 if $n^r_{m_k} \neq \text{null}$ and $\text{h}_{keys} > 4$ then
       17 $M \leftarrow M \cup \{ m_k \}$
     18 $m^r_k \leftarrow \arg \min_{m \in M'} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g \}$
     19 if $m_k = \text{null}$ then
       20 if $n^r_{m_k} - 1 > n^r_{m_k}$ or $n^r_{m_k} = \text{null}$ then
         21 $N_k \leftarrow n^r_{m_k} - 1$
       22 else
         23 $N_k \leftarrow n^r_{m_k} + 1$
       24 return $N_k$
     25 else
       26 return $m_k, \text{null}$
```

### 4.3 Inverse Queuing Model-Based Controller Design

If a mature reference performance $m^r_k$ can be found, feedback control is applied to follow the reference time $W^r_{m_k}$. The control error is

$$e_k = W^r_{m_k} - y_k.$$  

A controller should be designed based on performance model in Equation (5) and $e_k$. However, designing a feedback controller based on the hybrid non-linear performance model directly is very complex. Therefore, an inverse function of queuing model is used to linearize the performance model which simplifies the design of feedback controllers [19], [30] as shown in Fig. 3. Because it is complex to deduce
an inverse–queuing function from Equation (4) directly, an exhausted search method is used to implement the inverse–queuing function to find the corresponding container number \( N_k \) given average response time \( u_k \) (control input) as shown in Algorithm 2. According to queuing theory, the total processing rate \( N_k \times \mu \) should be larger than arrival rate \( \lambda \) to make system stable. Therefore, \( N_k \) is initialized to be \( \lfloor \frac{\lambda}{\mu} \rfloor \). Then \( N_k \) is increased one by one based on \( \lfloor \frac{\lambda}{\mu} \rfloor \) until \( W_s(N_k, \lambda, \mu) > u_k \). The queuing function from \( N_k \) to \( u_k \) (queuing model part) in the performance model is counteracted by the inverse–queuing function from \( u_k \) to \( N_k \). In other words, \( u'_k = u_k \) if queuing model part is accurate in describing multi-container systems. Then the linearized performance model is

\[
y_k = a \times y_{k-1} + (1 - a)u_{k-1} = a \times y_{k-1} + (1 - a)u_{k-1},
\]

and the Z-transfer function [29] of the performance model is

\[
Y \frac{U}{E} = \frac{1 - a}{z - a}.
\]

**Algorithm 2. Inverse Queuing Model (InverseQM)**

1. \( N_k \leftarrow \lfloor \frac{\lambda}{\mu} \rfloor \);
2. while \( W_s(N_k, \lambda, \mu) > u_k \) do
3. \( N_k \leftarrow N_k + 1 \);
4. return \( N_k \)

An integral feedback controller is designed based on the linear part merely ignoring the queuing model part as follows:

\[
u_k = u_{k-1} + K_I \times e_k.
\]

Z-transfer function of the integral controller is

\[
\frac{U}{E} = \frac{K_I z}{z - 1}.
\]

**4.4 Adaptive Output Error Mapping Method**

In order to keep the system stable, control input \( u_k \) should be smaller than \( W_s' = W_s(n^{r'}, \lambda, \mu) \) to make \( N_k \) larger than \( n^{r'} \). Meanwhile, \( u_k \) cannot be smaller than \( W_d = W_s(N_m, \lambda, \mu) \) to allocate no more than the maximum number of available containers \( N_m \). Because \( n^{r'} \) is determined by one sample and may be not accurate. To allow sampling on \( n^{r'} \), again, \( W_s' \) is set to be

\[
W_s' = \frac{W_s(n^{r'}, \lambda, \mu) + W_s(n_{m_k}^{r'}, 1, \lambda, \mu)}{2}.
\]

The output error \( e_k \) is used to adjust \( u_k \) by Equation (9). However, there are still unavoidable deviation between reference model \( m_k \) and profiled performance model as shown in Fig. 4. When the deviation is too large, \( u_k \) is likely to change greatly and exceed the lower or upper bounds \( [W_d', W_s'] \) of the queuing model part. Let \([W_d', W_s']\) be the lower and upper bounds of reference model \( m_k \) which are the smallest and largest response times smaller than \( W_{s_{max}} \), respectively. To guarantee the bounds \([W_d', W_s']\), an adaptive output-error mapping method (AOM) is proposed to map \( e_k \) from the reference model space to output error \( e_k' \) in the queuing model space.

**Theorem 1.** If output errors of the reference model are mapped to the performance model in proportional, there are

\[
e_k' = e_k \times K_a
\]

\[
K_a = \left\{ \begin{array}{ll}
\frac{u_{k-1} - W_d'}{y_{k-1} - W_{d_{m_k}}} & y_k > W_{d_{m_k}} \\
\frac{W_s' - u_{k-1}}{W_{s_{max}} - y_k} & \text{Otherwise}
\end{array} \right.
\]

**Proof 1.** It is assumed that \( u' \) (unknown) is the target reference point in queuing model space which is able to make the system get response time \( W_{s_{m_k}}' \). Then \( e_k' = u' - u_{k-1} \). The main objective of the mapping is to find an appropriate \( e_k' \) to make \( u_k \) move and more close to \( u' \). According to Equation (5), \( a \times y_{k-1} + (1 - a)u' \) and \( a \times y_{k-1} + (1 - a)u_{k-1} \) are corresponding values in the complete performance model (the total of queuing model and linear part) of \( u' \), \( W_d \) and \( u_{k-1} \) in the queuing model space, respectively. The output error in the performance model is

\[
e_k^p = (a \times y_{k-1} + (1 - a)u') - (a \times y_{k-1} + (1 - a)u_{k-1}).
\]

If values of the reference model is mapped to the performance model,
model in proportional, the ratio of \( e_k \) to \( y_k - W_d \) is equal to the ratio of \( e^p_k \) to \((a \times y_k - 1 + (1 - a)w_{k-1}) - m_k \times y_k - 1 + (1 - a)W_d\) as follows:

\[
\frac{e_k}{y_k - W_d} = \frac{e^p_k}{m_k} = \frac{(a \times y_k - 1 + (1 - a)w_{k-1}) - (a \times y_k - 1 + (1 - a)W_d)}{(1 - a)(w' - u_k - 1)} = \frac{e^p_k}{u_k - 1 - W_d}.
\]

When \( y_k \leq W^r \), the proof is similar.

In order to avoid large fluctuations, \( K_a \) is trimmed to 1 when \( K_a > 1 \). By replacing \( e_k \) of Equation (9) using \( e^p_k \), the integral controller becomes

\[
u_k = u_{k-1} + K_I \times e_k,
\]

where \( K_I = K_I \times K_a \) is called adaptive control gain.

### 4.5 Queuing-Length-Based Unstable State Scheduling

When the arrival rate \( \lambda \) is larger than the total processing ability \( \mu \), the system is unstable and Equations (2), (3) and (4) are not valid. Therefore, it is not suitable to find \( N_k \) based on \( e_k \) directly. In unstable states, the real queuing length \( L_\infty \) is larger than \( L_s(N_k, \lambda, \mu) \) and increases continually until the allowed maximum number of waiting connections is reached which represents the blocking degree. Therefore, a queuing-length-based unstable state provisioning method is applied as shown in Algorithm 3. First, if \( n_{s_k} > n^r_{m_k} \) which means \( N_k \) is a larger container number not fulfilling SLA, \( n^r_{m_k} \) is replaced by \( n_s \). Given \( N_k \) containers, the theoretical response time \( y_k \) of next step is the weighted combination of current response time \( y_k \) and expected response time \( W_s(N_k, \lambda, \mu) \) of queuing models according to Equation (5).

An iterative search is used to find the minimum number of containers \( N_k \) making \( y_k > W_{sla} \) based on \( \left[ \frac{y_k}{y_k} \right] \). When \( N_k \leq N_{k-1} \), at least one new container should be added every step. Each second, newly added \( N_k - N_{k-1} \) containers in this step can process additional \( \mu \times (N_k - N_{k-1}) \) requests. \( N_k \) is increased one by one to make sure that real-time queuing length \( L_r \) can decrease to theoretical queuing length \( L_s \) in \( T_r \) (e.g., 10) seconds by processing \( \mu \times (N_k - N_{k-1}) \times T_r \) additional requests on \( N_k - N_{k-1} \) containers. Larger \( T_r \) means decreasing queuing length more quickly. Finally, \( u_k \) is updated to guarantee that \( N_k \) containers are still rented in the next control step if there is no output errors.

### 4.6 Formal Description of Feedback_InverseQM

Formal description of Feedback_InverseQM is shown in Algorithm 4. At first, performance model is profiled using historical data. For queuing systems, when the real queuing length \( L_r \) is larger than theoretical queuing length \( L_s(N_k, \lambda, \mu) \), the system is unstable. Given arrival rate \( \lambda \), at least \( N = \left[ \frac{L_r}{\lambda} \right] \) containers are required to make system stable.

The theoretical response time given \( N \) containers is \( W_s(N, \lambda, \mu) \), which is called the largest stable response time. When \( y_k \) is larger than the largest stable response time, the system is unstable too. Because there are still deviations between the queuing model and real system, the queuing length is likely to be a little bit larger than the theoretical queuing length occasionally even if the system is stable. The above criterion of unstable is too strict, which is likely to lead to frequent switching between stable and unstable control making the system fluctuate greatly. Therefore, the system is judged to be unstable only when \( L_r \) is larger than \( \alpha \) (e.g., 10) times of \( L_s(N_k, \lambda, \mu) \) or \( y_k \) is larger than \( \beta \) (e.g., 1.2) times of \( W_s(N, \lambda, \mu) \). Giving larger \( \alpha \) and \( \beta \) means a more looser criterion of judging the system to be unstable. If the system is unstable, QLP is invoked. Otherwise, ARML is called to get a reference model \( m_k \) or obtain a sampling \( N_k \).

If \( m_k \neq null \), Equation (16) is used to obtain \( u_k \) and InverseQM is applied to get the real control action \( N_k \). Finally, the number of containers allocated to the Web system is adjusted to be \( N_k \).

**Algorithm 3. Queuing-Length-Based Provisioning (QLP)**

```
input: \( \lambda, \mu, W_{sla}, N_{k-1}, L_r, y_k \)
1 if \( n_s > n^r_{m_k} \) or \( n^r_{m_k} = null \) then
2 \( n^r_{m_k} \leftarrow n_s; \)
3 \( N_k \leftarrow \left[ \frac{n^r_{m_k}}{\mu} \right], T_r \leftarrow 10; \)
4 \( y_{k+1} \leftarrow a \times y_k + W_s(N_k, \lambda, \mu) \times (1 - a); \)
5 while \( y_{k+1} > W_{sla} \) do
6 \( N_k \leftarrow N_k + 1; \)
7 \( y_{k+1} \leftarrow a \times y_k + W_s(N_k, \lambda, \mu) \times (1 - a); \)
8 if \( N_k \leq N_{k-1} \) then
9 \( N_k \leftarrow N_{k-1} + 1 \)
10 while \( \mu \times (N_k - N_{k-1}) \times T_r < L_r - L_s(N_k, \lambda, \mu) \) do
11 \( N_k \leftarrow N_k + 1; \)
12 if \( (N_k - 1) \times \mu > \lambda \) then
13 \( u_k \leftarrow W_s(N_k, \lambda, \mu) + \left[ \frac{2}{\mu} \right]; \)
14 else
15 \( u_k \leftarrow W_s(N_k, \lambda, \mu) + \epsilon, \epsilon \) is infinitely small; 
16 return \( N_k \)
```

**Algorithm 4. Inverse Queuing Model Based Feedback Control (Feedback_InverseQM)**

```
input: \( \lambda, \mu, s, y_k \)
1 Initialize \( \alpha \leftarrow 10, \beta \leftarrow 1.2, y_k \leftarrow y_k; \)
2 \( \mu_k, c, a \leftarrow \) Profiling the performance model;
3 \( \mu \leftarrow \mu_k + c/\lambda, N \leftarrow \left[ \frac{\lambda}{\mu} \right]; \)
4 if \( L_r > L_s(N_k, \lambda, \mu) \times a \) or \( y_k > W_s(N, \lambda, \mu) \times \beta \) then
5 \( N_k \leftarrow \text{QLP}(\lambda, \mu, W_{sla}, N_{k-1}, L_r, y_k); \)
6 else
7 \( m_k, N_k \leftarrow \text{ARML}(s, \lambda, m_k, W_{sla}, y_k); \)
8 if \( m_k \neq null \) then
9 \( u_k \leftarrow \) Equation (16); 
10 \( N_k \leftarrow \text{InverseQM}(\lambda, \mu, u_k); \)
11 Allocate \( N_k \) containers to the system;
```

### 5 PERFORMANCE EVALUATION

Our proposed Feedback_InverseQM is implemented as a user-level scheduler for Kubernetes. It is compared with
existing algorithms on a real Kubernetes cluster which locates on four physical machines with the configuration of 6~12 virtual CPU cores and 8~16 GB Memory. The Kubernetes cluster consists of one Master and four Worker nodes. A service for calculating Fibonacci numbers is adopted as the test-bed and the input of the service is the length of the generated Fibonacci series which is selected from the interval [26,33] for each request randomly. The SLA is 0.1 second. Requests of the service are redirected to different containers by the nginx-ingress-controller using exponentially weighted moving average (EWMA) as the load-balancing algorithm [36]. The response time of every request is stored in the log of nginx, and ASC obtains the average response time of every minute by reading logs through the java client-interface of Kubernetes (JCI). The queuing length and request arrival rate are obtained by reading status information of nginx using the http protocol. Pod auto-scaling commands generated by ASC are sent to Kubernetes by changing the value of deployment’s replicas through JCI. The connection timeout time of nginx is 10 seconds and the allowed maximum number of connections is 500 per container. The user access traces of Wikipedia [38] as shown in Fig. 5 with common peaks and valleys of Web systems are used to generate requests through JMeter [39].

Feedback_InverseQM is first compared with Feedback_QMDL which is a classical feedback control method for QoS control based on queuing-model-derived linear models [17], [18]. Although Feedback_QMCA [30] is tailored for hourly-priced VM provisioning, Feedback_InverseQM is still compared with Feedback_QMCA by removing the VM-releasing status checking. Finally, Feedback_InverseQM is also compared with Feedback_InverseP [19] which is one of the elastic container provisioning algorithms considering QoS control. Average response times and the total consumed CUs are metrics of algorithm comparison. The length of control interval of all algorithms is 250 seconds. Since Feedback_InverseQM has an initial sampling period, the cost of each compared algorithm is only the accumulation of consumed CUs after the initial 500 minutes for fair comparison.

5.1 Parameter Tunning

The control gain $K_I$ determines the poles of the system which has a great impact on the settle time and overshoot. Poles can be derived by setting $z^2 + [(1 - a)K_I - 1 - a]z + a = 0$ given $K_I$. For example, poles $p_1 = 0.44 + 0.32619013j$ and $p_2 = 0.44 - 0.32619013j$ when $K_I = 0.6$. According to root locus which draws the figure of poles as $K_I$ changes [29], larger $K_I$ usually means larger overshoots, and too small or too large $K_I$ are likely to incur long settle times. A set of candidate values $S_{K_I} = \{0.05, 0.1, 0.15, 0.3, 0.6\}$ are selected based on root locus. Then the real performance of $K_I \in S_{K_I}$ is evaluated by experiments. Figs. 6 and 7 show consumed container numbers and average response times of Feedback_InverseQM with different $K_I$ which illustrate that container numbers of Feedback_InverseQM changes more quickly as $K_I$ increases in total. For smaller $K_I$, Feedback_InverseQM reacts to excess containers and SLA violations very slowly. For example, excess containers cannot be released in time consuming the most cost (2454 CUs) when $K_I = 0.05$ and SLA is violated for many periods when $K_I = 0.1$. On the contrary, for larger $K_I$, the container number fluctuate fiercely leading to more SLA violations when $K_I \geq 0.3$ and extremely large $K_I = 0.6$ even incurs the second most cost (2290 CUs) because of frequent container allocating and deallocating. Therefore, $K_I = 0.15$ is finally selected which obtains the most appropriate control strength leading to fewer SLA violations and a lower cost simultaneously.
According to the performance tuning results of [30], poles of Feedback_QMDL and Feedback_QMCA are set to be 0.9 and 0, respectively. The pole of Feedback_InverseP is set to be 0.95 consistent with [19].

To obtain values of performance model’s parameters, the least square method is applied based on historical data (including average response times, container numbers and arrival rates) collected from Kubernetes platforms under $0 < a < 0.3$, $\mu_b > 0$ and $c > 0$ constraints. Without constraint $0 < a < 0.3$, $\alpha > 0.9$ is likely to be obtained because of the similarity between two consecutive response times which will suppress the impact of the queuing model part. Constraint $c > 0$ is used to guarantee that the processing ability decreases as the arrival rate increases. If $\mu_b$ of the trained model is smaller than zero, processing ability $\mu$ will be smaller than zero when $\lambda > \frac{1}{\mu_b}$ which makes the model meaningless. Therefore, $\mu_b > 0$ is necessary. For our kubernetes-based platform, $\mu_b = 7.771$, $c = 1574.510$ and $a = 0.215$ are obtained based on the given historical data which may change over time. Parameters of the fixed-processing-rate-based performance model can be acquired similarly by setting $c = 0$. Fig. 8 shows real response times and estimated values of profiled performance models which illustrates that the proposed varying-processing-rate-based method (blue dots) describes the system more accurately than the fixed-request-processing-rate based method (green dots).

### 5.2 Experimental Results

Table 3 shows percentages of SLA violations and costs of compared algorithms which illustrate that proposed FeedBack_InverseQM obtains the lowest percentage of SLA violation (8.44 percent smaller than that of the best existing algorithm) with the second lowest cost (2250 CUs) in total.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>SLA Violations</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback_InverseQM</td>
<td>2.36%</td>
<td>2250 CUs</td>
</tr>
<tr>
<td>Feedback_QMDL</td>
<td>10.80%</td>
<td>2289 CUs</td>
</tr>
<tr>
<td>Feedback_QMCA</td>
<td>21.26%</td>
<td>1989 CUs</td>
</tr>
<tr>
<td>Feedback_InverseP</td>
<td>52.99%</td>
<td>2487 CUs</td>
</tr>
</tbody>
</table>

Fig. 9 shows container numbers and response times of FeedBack_InverseQM which denotes that most response times are smaller than $W_{sla}$. Reasons of FeedBack_InverseQM’s best performance are as follows. First, ARML of FeedBack_InverseQM is helpful to improving the accuracy of output errors. Because different arrival rates have different stable points, samples are collected by ARML to study reference response times for diverse arrival rates leading to some fluctuations in the initial stage of Fig. 9. Fig. 8 shows stable points of two mature reference models with different arrival rates. In Fig. 10, the studied reference time changes over time which increases the accuracy of output errors. Second, AOM of FeedBack_InverseQM is able to improve control stability. FeedBack_InverseQM is sensitive to output errors when $u_{k-1}$ is near relatively flatten parts of the queuing model as shown in Fig. 4. Flatten parts of performance models reflect the nature of queuing systems which enable controllers react to SLA violations quickly. However, there are unavoidable deviations between the reference model and the profiled queuing model. Sometimes, the original output errors based on the reference model are
so large which makes $u_k$ fluctuate drastically. Fig. 11 shows adaptive control gains of different steps generated by AOM which are used to map output errors from the reference model to the queuing model in proportion to avoid such fierce fluctuations. Both ARML and AOM make FeedBack_InverseQM obtain the most stable performance as shown in Fig. 9 except the initial sampling stage.

Fig. 12 illustrates that FeedBack_QMDL reacts slowly to fast arrival-rate changes leading to SLA violations or delaying the release of excess containers. The reason is that the output of FeedBack-QMDL is the plus of M/M/N model’s output and an additional value determined by feedback control. Meanwhile, different arrival rates need quite different additional values, and the changing speed of the additional value cannot meet the requirement when the arrival rate changes quickly. However, the changing speed of the additional value cannot be increased by setting smaller poles (e.g., 0.6) anymore, because it is likely to allocate or release excess containers incurring more SLA violations or higher costs for scenarios with slow arrival rate changing speeds. The main reason is that the changing speed of the additional value is fixed and linear to the output errors without considering various arrival rates with different changing speeds.

Fig. 13 demonstrates that FeedBack_QMCA is very likely to allocate or deallocate excess containers leading to frequent large fluctuations at periods with slow arrival-rate changing speeds of which the reasons are as follows. In FeedBack_QMCA, the inaccuracy of queuing model is fixed by an arrival-rate adjustment coefficient. Experimental results illustrate that different arrival rates need quite different adjustment coefficients. When the arrival rate changes quickly, it takes a long time to adjust the coefficient leading to SLA violations or higher costs given small control gains. Therefore, the current proportional control gain has been increased as large as possible to speed up the changing speed of the adjustment coefficient to meet the requirement of fast arrival rate changes. However, the controller gain, which fulfills the fast-changed arrival rates, makes the system fluctuate when arrival rates change slowly. It is hard to find an appropriate gain suitable for different arrival-rate changing speeds. Meanwhile, a fixed reference point is not able to generate suitable output errors for all arrival rates which misleads the controller to release or rent excess containers incurring SLA violations or higher costs.

Fig. 14 shows that the container number of FeedBack_InverseP fluctuates fiercely and SLA is violated frequently although a large pole of 0.95 (long settle times) has been given. Both FeedBack-QMDL and FeedBack_QMCA use a gradually
changed additional value or arrival-rate coefficient to amend the inaccuracy of queuing models which avoids fierce fluctuation of container numbers. On the contrary, similar with FeedBack_InverseQM, FeedBack_InverseP’s inverse-proportional performance model is also very sensitive to output errors when $\mu_{l-1}$ is near the relatively flatter parts of the performance model. However, a fixed reference point and inconsistency between the inverse-proportional performance model and the real system cannot always generate appropriate output errors for different arrival rates incurring fierce fluctuations of container numbers.

6 CONCLUSION AND FUTURE WORK

In this paper, an inverse-queuing-model-based feedback control method has been proposed to provision containers to Web systems in Kubernetes-based platforms elastically for guaranteeing QoS. Experimental results show that the hybrid of varying-processing-rate-based queuing model and linear model is able to increase the accuracy of performance model. Meanwhile, automatic reference-model learning and adaptive output-error mapping increase the accuracy of output errors which decreases the percentage of SLA-violation by 8.44 percent and obtains the second lowest cost. Designing container auto-scaling algorithms considering geography distribution of Cloud, Fog and Edge resources is promising future work.

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